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ScienceDirect
International Journal of Heat and Mass Transfer 50 (2007) 1902–1908
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer

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Received 20 July 2005
Available online 18 December 2006

Abstract

Analytical closed-form solution of the unsteady hydro-magnetic natural convection heat and mass transfer flow of a rotating, incompressible, viscous Boussinesq fluid is presented in this study in the presence of radiative heat transfer and a first order chemical reaction between the fluid and the diffusing species. The Rosseland approximation for an optically thick fluid is invoked to describe the radiative flux. Results obtained show that a decrease in the temperature boundary layer occurs when the Prandtl number and the radiation parameter are increased and the flow velocity approaches steady state as the time parameter t, is increased. These findings are in quantitative agreement with earlier reported studies.

Keywords: Natural convection; Rotating fluid; Radiative heat transfer; Optically thick fluid

1. Introduction

The flow of an incompressible Boussinesq fluid in the presence of rotation has applications in space science and engineering fluid dynamics. Bestman and Adjepong [1] studied the unsteady hydro-magnetic free convection flow with radiative heat transfer in a rotating fluid. Jha [2] studied MHD free convection and mass transfer flow through a porous medium but did not consider the effect of radiation which is of great relevance to astrophysical and cosmic studies. The effects of Hall current on hydro-magnetic free convection with mass transfer in a rotating fluid was studied by Agrawal et al. [3]. Singh and Sacheti [4] presented a study on the finite difference analysis of unsteady hydro-magnetic free convection flow with constant heat flux, while Ram and Jain [5] presented the result of a study on hydro-magnetic Ekman layer on convective heat generating fluid in slip flow regime. Helmy [6] focused on MHD flow in a micro-polar fluid. Recently, Chamkha [7] investigated unsteady convective heat and mass transfer past a semi-infinite permeable moving plate with heat absorption where it was found that increase in solutal Grashoff number enhanced the concentration buoyancy effects leading to an increase in the velocity. In another recent study Ibrahim et al. [8] investigated unsteady magneto-hydrodynamic micro-polar fluid flow and heat transfer over a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. Chamkha and Cookey [7,9], give a good review on MHD flows through a porous medium.

Some non-Newtonian fluids with large absorption coefficient are classified as optically thick, Bestman [16]. The aim of the present study, which deals with optically thick fluids with large absorption coefficient, is to analyze the effects of radiation on unsteady three-dimensional natural convection involving heat and mass transfer for heating of the porous plate ($G_r < 0$) by free convection currents,

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doi:10.1016/j.ijheatmasstransfer.2006.10.016

Nomenclature

u, v	velocity components
q	complex velocity
U_0	scale of free stream velocity
z	coordinate
ρ	fluid density
T	dimensional temperature
c	dimensional species concentration
t	time
k	thermal conductivity
D_m	solutal diffusivity
ρ	density
c_p	specific heat at constant pressure
ε	time corrective parameter ($\varepsilon \ll 1$)
β and β^*	coefficients of volume expansion due to temperature and concentration
μ	permeability
M	magnetic Hartmann number
Pr	Prandtl number
Sc	Schmidt number
k_r	chemical reaction constant
K	porosity parameter
σ^*	Stefan–Boltzmann constant
α^2	absorption coefficient
λ	wavelength
B	Planck's constant
θ	non-dimensional temperature
C	non-dimensional species concentration
g	acceleration due to gravity
G_r	free convection parameter
G_c	modified free convection parameter
σ	electrical conductivity
ν	kinematic coefficient of viscosity
Ω	angular velocity
H_0	magnetic field strength
q_r	radiative flux vector

Superscript

$'$	differentiation with respect to z

Subscripts

w	wall condition
∞	free stream condition

and for cooling of the porous plate ($G_r > 0$) which should increase the applicability of the study reported in Ogulu and Cookey [10] especially with the incorporation in this study of the effect of chemical reaction rate.

2. Governing equations and their solution

We consider in three dimensions the unsteady motion of an incompressible electrically conducting viscous fluid which moves in its own plane with velocity U_0 and rotates with angular velocity Ω as in [1]. We assume a uniform magnetic field B_0 applied in the direction of the flow fixed relative to the plate. We also assume that induced magnetic fields are negligible in comparison with the applied field. Further, we assume no applied voltage present which means no electric field present and viscous dissipation heating is absent in the energy equation. With these assumptions and those usually associated with the Boussinesq approximations, the proposed governing equations are

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = \nu \frac{\partial^2 u'}{\partial z'^2} - \frac{\sigma B_0^2 u'}{\rho_\infty} - \frac{\nu u'}{K'} + g\beta(T - T_\infty) + g\beta^*(c - c_\infty) \quad (1)$$

$$\frac{\partial v'}{\partial t'} - 2\Omega' u' = \nu \frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma \mu^2 H_0^2 v'}{\rho_\infty} \quad (2)$$

$$\rho_\infty c_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial z'^2} - \frac{\partial q_r'}{\partial z'} \quad (3)$$

$$\rho_\infty c_p \frac{\partial c'}{\partial t'} = D_m \frac{\partial^2 c'}{\partial z'^2} - k_r' c' \quad (4)$$

subject to the conditions

$$\begin{aligned} &u' = U_0,\ v' = 0,\ T = T_w,\ c' = c_w \quad \text{on } z' = 0 \\ &u' = 0,\ v' = 0,\ T = T_\infty,\ c' = c_\infty \quad \text{as } z' \to \infty \end{aligned} \quad (5)$$

We now introduce the following non-dimensional quantities and parameters

$$\begin{aligned} &t' = \frac{\nu t}{U_0^2}, \quad z' = \frac{\nu z}{U_0}, \quad u' = uU_0, \quad v' = vU_0, \\ &K' = \frac{\nu^2 K}{U_0^2}, \quad \Omega' = \frac{U_0^2 \Omega}{\nu}, \quad q_r' = \frac{U_0 k q_r}{\rho c_p}, \quad Sc = \frac{\mu c_p}{D_m}, \\ &Pr = \frac{\mu c_p}{k}, \quad M^2 = \frac{\sigma \mu \nu H_0^2}{\rho_\infty U_0^2}, \quad G_r = \frac{g\beta\nu(T - T_\infty)}{U_0^3}, \\ &G_c = \frac{g\beta\nu(c - c_\infty)}{U_0^3}, \quad \theta = \frac{T - T_w}{T - T_\infty}, \quad C = \frac{c - c_w}{c - c_\infty} \end{aligned} \quad (6)$$

Introducing Eq. (6) into Eqs. (1)–(5) we obtain

$$\frac{\partial u}{\partial t} - 2\Omega v = \frac{\partial^2 u}{\partial z^2} - M^2 u - \frac{u}{K} + G_r\theta + G_c C \quad (7)$$

$$\frac{\partial v}{\partial t} - 2\Omega u = \frac{\partial^2 v}{\partial z^2} - M^2 v \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial z^2} - \frac{\partial q_r}{\partial z}\right) \quad (9)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial z^2} - k_r C \quad (10)$$

We now find it convenient to combine Eqs. (1) and (2) into a single equation. We multiply Eq. (2) by i and add the resultant to Eq. (1) to obtain

$$\frac{\partial q}{\partial t}+\left(2\mathrm{i}\Omega+M^2+\frac{1}{K}\right)q=\frac{\partial^2 q}{\partial z^2}+G_r\theta+G_cC \tag{11}$$

$q=u+\mathrm{i}v$ and $\mathrm{i}=\sqrt{-1}$. Further, for the radiative heat flux in Eq. (9) we invoke the differential approximation, Elbarbary and Elgazery [11]

$$\nabla\cdot q_r=4(T-T_w)\int_0^\infty \alpha^2\left(\frac{\partial B}{\partial T}\right)\mathrm{d}\lambda \tag{12}$$

For an optically thick fluid, as noted in Azzam [15], in addition to emission there is also self-absorption and usually the absorption coefficient is wavelength dependent and large (as noted in [16]) so we can adopt the Rosseland approximation of Eq. (12) where the radiative flux vector q_r is given by

$$q_r=-\frac{4\sigma^*}{3\alpha}\frac{\partial T^4}{\partial z} \tag{13}$$

Assuming small temperature differences within the flow we can expand T in a Taylor series about a free stream temperature T_∞, neglecting higher order terms, we obtain

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \tag{14}$$

Substituting Eq. (14) into Eq. (9) we obtain

$$\frac{\partial\theta}{\partial t}=\left(\frac{1+N}{Pr}\right)\frac{\partial^2\theta}{\partial z^2} \tag{15}$$

where $N=\frac{3\alpha k}{4\sigma^*T_\infty^3}$ is the radiation parameter. The initial and boundary conditions are now

$$\begin{aligned}&t\leqslant 0: q(z,t)=\theta(z,t)=C(z,t)=0\\&t>0:\{q(0,t)=q_0\quad \theta(0,t)=1\quad C(0,t)=1\\&\qquad\ \{q(\infty,t)\to 0\quad \theta(\infty,t)\to 0\quad C(\infty,t)\to 0\end{aligned} \tag{16}$$

The mathematical statement of the problem is now complete.

The solution of Eqs. (10) and (15) employing Laplace transform technique subject to the conditions in Eq. (16) can be put in the form

$$\theta(z,t)=\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{a}{t}}\right) \tag{17}$$

and

$$C(z,t)=\frac{1}{2}\left\{\mathrm{e}^{z\sqrt{k_rSc}}\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{Sc}{t}}+\sqrt{k_rt}\right)+\mathrm{e}^{-z\sqrt{k_rSc}}\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{Sc}{t}}-\sqrt{k_rt}\right)\right\} \tag{18}$$

where $a=\frac{Pr}{1+N}$, and subsequently $h=\frac{1}{Sc}$.

We now substitute Eqs. (16) and (17) into Eq. (11) and take the Laplace transform of the resultant equation. On appeal to Abramowitz and Stegun [12] for inverse transforms and convolution, we can show that for

(i) Constant motion:

$$\begin{aligned}q(z,t)=&\,q_0\left\{\mathrm{e}^{-z\sqrt{d}}\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{1}{t}}-\sqrt{dt}\right)+\mathrm{e}^{z\sqrt{d}}\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{1}{t}}+\sqrt{dt}\right)\right\}\\&-\frac{G_r^*}{d^*}\left\{\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{a}{t}}\right)-\mathrm{e}^{d^*t}\left[\mathrm{e}^{-z\sqrt{d^*}}\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{a}{t}}-\sqrt{d^*t}\right)\right.\right.\\&\left.\left.+\mathrm{e}^{z\sqrt{d^*}}\mathrm{erfc}\left(\frac{z}{2}\sqrt{\frac{a}{t}}+\sqrt{d^*t}\right)\right]\right\}\\&-\frac{G_c^*}{d^*}\left\{\mathrm{e}^{-k_rz}\mathrm{erfc}\left(\frac{z}{2\sqrt{ht}}-k_r\sqrt{t}\right)+\mathrm{e}^{k_rz}\mathrm{erfc}\left(\frac{z}{2\sqrt{ht}}+k_r\sqrt{t}\right)\right.\\&-\mathrm{e}^{h^*t}\left[\mathrm{e}^{-zk_r\sqrt{h^*}}\mathrm{erfc}\left(\frac{z}{2\sqrt{ht}}-k_r\sqrt{h^*t}\right)\right.\\&\left.\left.+\mathrm{e}^{k_rz\sqrt{h^*}}\mathrm{erfc}\left(\frac{z}{2\sqrt{ht}}+k_r\sqrt{h^*t}\right)\right]\right\}\end{aligned} \tag{19}$$

$$d=(M^2+1/K)+2\mathrm{i}\Omega,\quad d^*=\frac{d}{a-1},$$
$$h^*=\frac{k_r-hd}{h-1},\quad G_r^*=\frac{G_r}{1-a},\quad G_c^*=\left(\frac{h}{h-1}\right)G_c$$

(ii) Decaying oscillatory motion:

We can express the velocity field as

$$q(z,t)=\Phi(z,t)+A_j(z,t) \tag{20}$$

Such that

$$\Phi(z,t)=\int_0^t\Phi_*(z,\tau)f(t-\tau)\mathrm{d}\tau \tag{21}$$

where

$$\Phi_*(z,t)=\frac{H(t)}{2\sqrt{\pi}}z\tau^{-3/2}\mathrm{e}^{-\mathrm{d}\tau}-\frac{z^2}{4\tau} \tag{22}$$

$H(t)$ is Heaviside unit function. In this case we take $f(t)$ as

$$f(t)=\frac{H(t)}{2}\left[\mathrm{e}^{-\left(\lambda_1^2-\mathrm{i}\omega\right)t}+\mathrm{e}^{-\left(\lambda_1^2+\mathrm{i}\omega\right)t}\right] \tag{23}$$

where λ_1 and ω are both real positive dimensionless constants. Substitution of Eq. (22) into Eq. (20) gives

$$\begin{aligned}q(z,t)=&\frac{H(t)}{4}\left\{\mathrm{e}^{-\left(\lambda_1^2-\mathrm{i}\omega\right)t}\int_0^t\frac{z}{2\sqrt{\pi}}\tau^{-3/2}\mathrm{e}^{-\left(d-\lambda_1^2+\mathrm{i}\omega\right)\tau}-\frac{z^2}{4\tau}\mathrm{d}\tau\right\}\\&+\frac{H(t)}{4}\left\{\mathrm{e}^{-\left(\lambda_1^2+\mathrm{i}\omega\right)t}\int_0^t\frac{z}{2\sqrt{\pi}}\tau^{-3/2}\mathrm{e}^{-\left(d-\lambda_1^2-\mathrm{i}\omega\right)\tau}-\frac{z^2}{4\tau}\mathrm{d}\tau\right\}\end{aligned} \tag{24}$$

Let $\alpha_1\pm\mathrm{i}\beta_1\equiv\left[\left(M^2+\frac{1}{K}-\lambda_1^2\right)+\mathrm{i}(\omega+2\Omega)\right]^{1/2}$ and

$$\alpha_2\pm\mathrm{i}\beta_2\equiv\left[\left(M^2+\frac{1}{K}-\lambda_1^2\right)+\mathrm{i}(2\Omega-\omega)\right]^{1/2}$$

Then when $\omega<2\Omega$ we have,

$$\alpha_1,\beta_1=\frac{1}{\sqrt{2}}\left\{\left[\left(M^2+\frac{1}{K}-\lambda_1^2\right)^2+(\omega+2\Omega)^2\right]^{1/2}+\left(M^2+\frac{1}{K}-\lambda_1^2\right)\right\}^{1/2}$$

and when $\omega \geqslant 2\Omega$

$$\alpha_2, \beta_2 = \frac{1}{\sqrt{2}}\left\{\left[\left(M^2+\frac{1}{K}-\lambda_1^2\right)^2+(\omega-2\Omega)^2\right]^{1/2} - \left(M^2+\frac{1}{K}-\lambda_1^2\right)\right\}^{1/2}$$

Hence

$$\begin{aligned} q(z,t) = {} & \frac{H(t)}{4}\mathrm{e}^{-\left(\lambda_1^2-\mathrm{i}\omega\right)t}\left\{\mathrm{e}^{-z(\alpha_1+\mathrm{i}\beta_1)}\operatorname{erfc}\left(\frac{z-2(\alpha_1+\mathrm{i}\beta_1)t}{2\sqrt{t}}\right) + \mathrm{e}^{z(\alpha_1+\mathrm{i}\beta_1)}\operatorname{erfc}\left(\frac{z+2(\alpha_1+\beta_1)t}{2\sqrt{t}}\right)\right\} \\ & + \frac{H(t)}{4}\mathrm{e}^{-\left(\lambda_1^2+\mathrm{i}\omega\right)t}\left\{\mathrm{e}^{-z(\alpha_2\pm\mathrm{i}\beta_{21})}\operatorname{erfc}\left(\frac{z-2(\alpha_2\pm\mathrm{i}\beta_2)t}{2\sqrt{t}}\right) + \mathrm{e}^{z(\alpha_2\pm\mathrm{i}\beta_2)}\operatorname{erfc}\left(\frac{z+2(\alpha_2\pm\beta_2)t}{2\sqrt{t}}\right)\right\} \\ & - \frac{G_r^*}{d^*}\left\{\operatorname{erfc}\left(\frac{z}{2}\sqrt{\frac{a}{t}}\right) - \mathrm{e}^{d^*t}\left[\mathrm{e}^{-z\sqrt{d^*}}\operatorname{erfc}\left(\frac{z}{2}\sqrt{\frac{a}{t}}-\sqrt{d^*t}\right)\right] + \mathrm{e}^{z\sqrt{d^*}}\operatorname{erfc}\left(\frac{z}{2}\sqrt{\frac{a}{t}}+\sqrt{d^*t}\right)\right\} \\ & - \frac{G_c^*}{d^*}\left\{\mathrm{e}^{-k_r z}\operatorname{erfc}\left(\frac{z}{2\sqrt{ht}}-k_r\sqrt{t}\right) + \mathrm{e}^{k_r z}\operatorname{erfc}\left(\frac{z}{2\sqrt{ht}}+k_r\sqrt{t}\right)\right. \\ & \left. - \mathrm{e}^{h^*t}\left[\mathrm{e}^{-zk_r\sqrt{h^*t}}\operatorname{erfc}\left(\frac{z}{2\sqrt{ht}}-k_r\sqrt{h^*t}\right) + \mathrm{e}^{k_r z\sqrt{h^*}}\operatorname{erfc}\left(\frac{z}{2\sqrt{ht}}+k_r\sqrt{h^*t}\right)\right]\right\} \end{aligned} \quad (25)$$

Having obtained the expression for the velocity field one can now proceed to calculate the complex non-dimensional skin friction. For the constant motion we have

$$\tau_x + \mathrm{i}\tau_y = q_0\left\{-2\sqrt{d}\operatorname{erfc}\left(\sqrt{dt}\right) - \mathrm{e}^{-dt}\sqrt{\frac{1}{t}}\right\} + \tau_{0j}, \quad j = 1, 2 \quad (26)$$

and for the decaying oscillatory motion the complex non-dimensional skin friction is given as

$$\begin{aligned} \tau_x + \mathrm{i}\tau_y = {} & \frac{H(t)}{2}\mathrm{e}^{-\left(\lambda^2-\mathrm{i}\omega\right)t}(\alpha_1+\mathrm{i}\beta_1)\operatorname{erfc}\left[(\alpha_1+\mathrm{i}\beta_1)-\sqrt{t}\right] \\ & - \frac{H(t)}{2}\mathrm{e}^{-\left(\lambda^2+\mathrm{i}\omega\right)t}(\alpha_2\pm\mathrm{i}\beta_2)\operatorname{erfc}\left[(\alpha_2\pm\mathrm{i}\beta_2)-\sqrt{t}\right] \\ & - \frac{H(t)\mathrm{e}^{-\mathrm{d}\tau}}{\sqrt{\pi t}} + \tau_{0j} \end{aligned} \quad (27)$$

where

$$\tau_{01} = -\frac{G_r^*}{d^*}\left\{2\mathrm{e}^{d^*t} - \frac{1}{2}\sqrt{\frac{a}{t}}\mathrm{e}^{2d^*t}\right\}$$

and

$$\tau_{02} = -\frac{G_c^*}{d^*}\left\{-2k_r\operatorname{erfc}\left(k_r\sqrt{t}\right) - \frac{1}{\sqrt{ht}}\mathrm{e}^{k_r^2 t} + k_r\mathrm{e}^{h^*t}\sqrt{h^*}\operatorname{erfc}\left(k_r\sqrt{h^*t}\right) - \frac{1}{\sqrt{ht}}\mathrm{e}^{-\left(k_r^2-1\right)h^*t}\right\}$$

τ_{01} corresponds to the complex non-dimensional skin friction when $\omega < 2\Omega$, while τ_{02} will correspond to the complex non-dimensional skin friction when $\omega \geqslant 2\Omega$.

3. Results and discussion

The problem of radiative heat transfer to unsteady hydro-magnetic flow involving heat and mass transfer is addressed in this study. Numerical calculations have been carried out for the non-dimensional temperature θ, concentration C, complex velocity q and skin friction τ taking the Heaviside step function $H(t) = 1$, for simplicity, keeping the other parameters of the problem fixed. The solution obtained for the velocity is complex and only the real part of the complex quantity is invoked for the numerical discussion with the help of Abramowitcz and Stegun [12]. Numerical calculations of these results are presented graphically in Figs. 1–10. These results show the effect of material parameters on the temperature distribution, concentration profiles, complex velocity and the shear stress at the wall.

In Fig. 1 we depict the temperature distribution for $Pr = 0.71$, as corresponds to air and $Pr = 7.0$, as corresponds to water at room temperature and one atmosphere pressure, highlighting the effect of radiation. We observe a decrease in the temperature and the temperature boundary layer as the radiation parameter N, is increased and also as the Prandtl number Pr, increases. This second observation agrees with the report in Kim [17] where it was observed that this decrease in the temperature boundary layer is accompanied with a more uniform temperature distribution across the boundary layer. The temperature is observed to decrease steeply and exponentially away from

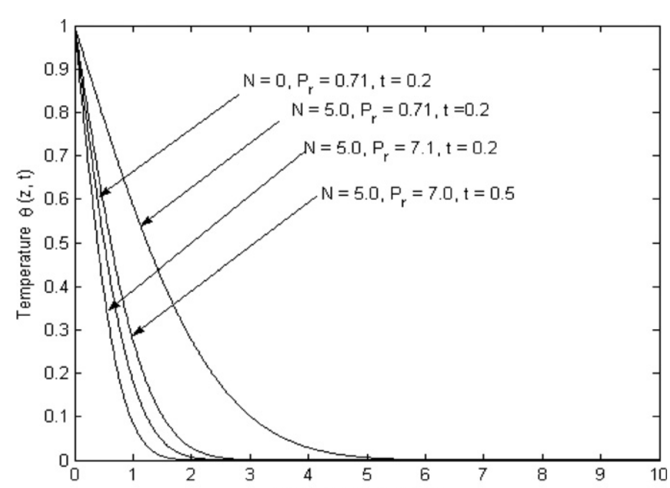

Fig. 1. Temperature profiles.

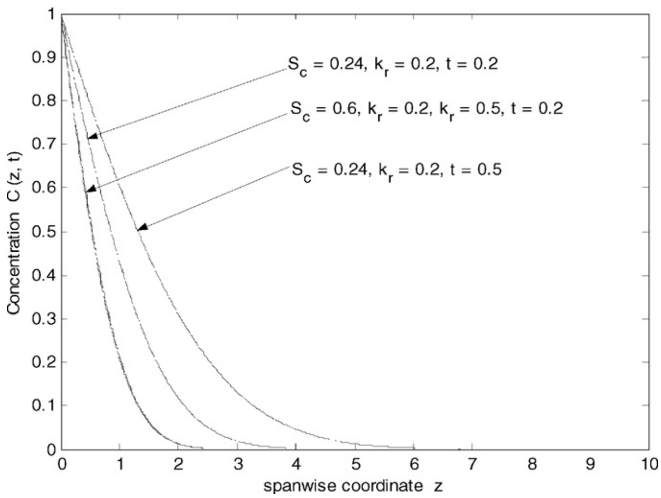

Fig. 2. Concentration profiles.

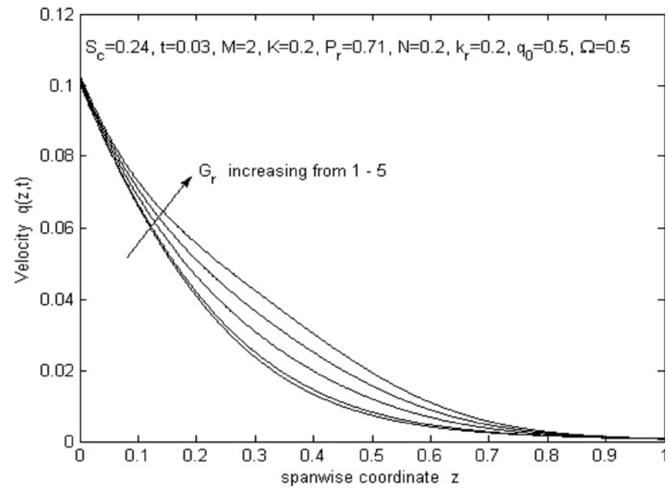

Fig. 5. Velocity profiles showing the effect of the free convection parameter G_r, $t = 0.03$.

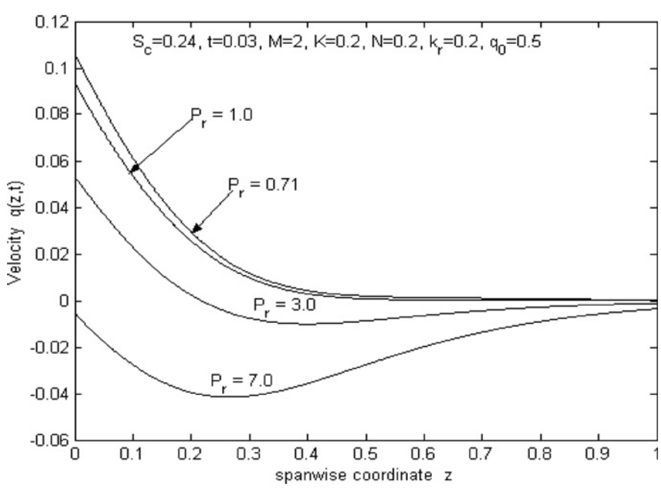

Fig. 3. Velocity profiles showing the effect of Prandtl number.

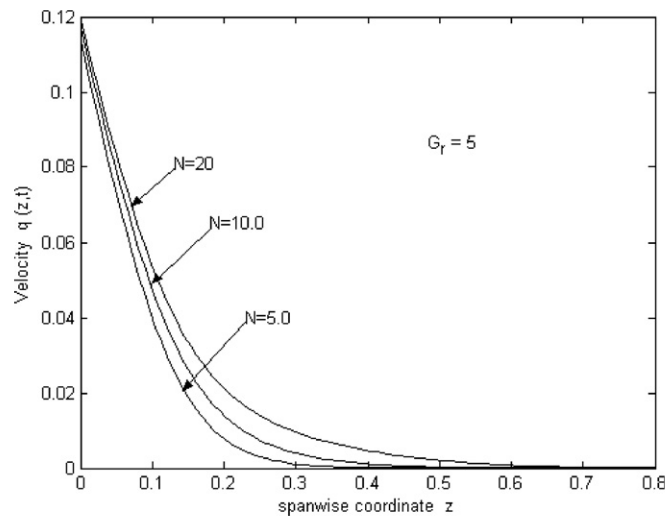

Fig. 6. Effect of radiation on the velocity profile; $G_r = 5$, $t = 0.03$.

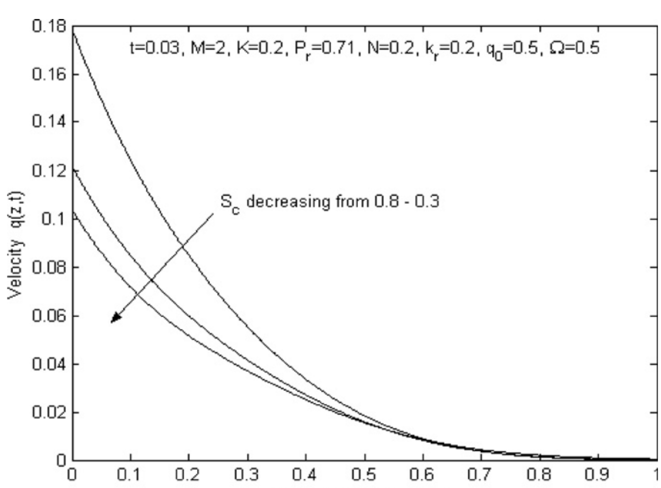

Fig. 4. Velocity profiles showing the effect of the Schmidt number.

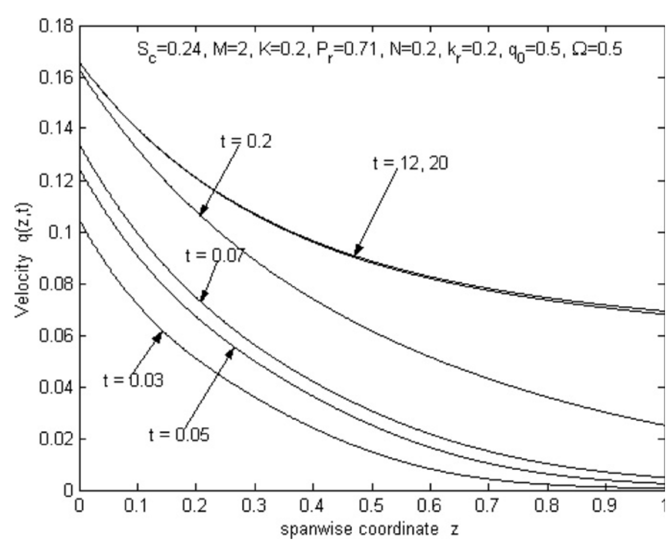

Fig. 7. Velocity profiles showing the effect of time parameter, t.

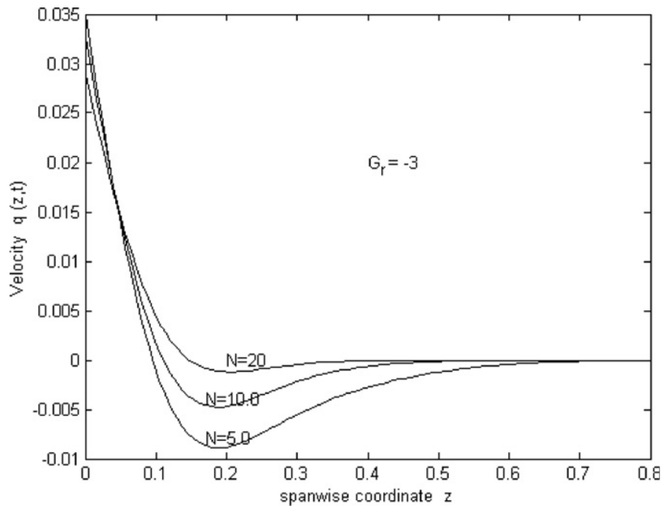

Fig. 8. Effect of radiation on the velocity distribution; $G_r = -3$, $t = 0.03$.

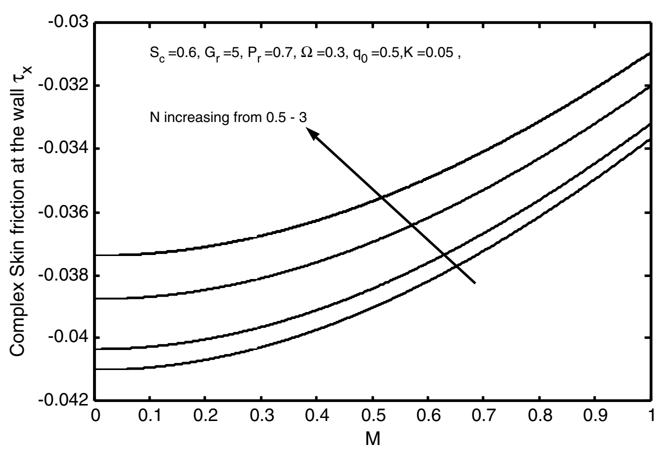

Fig. 9. Effect of radiation on the real part of the skin friction when $\omega < 2\Omega$, and $t = 0.03$.

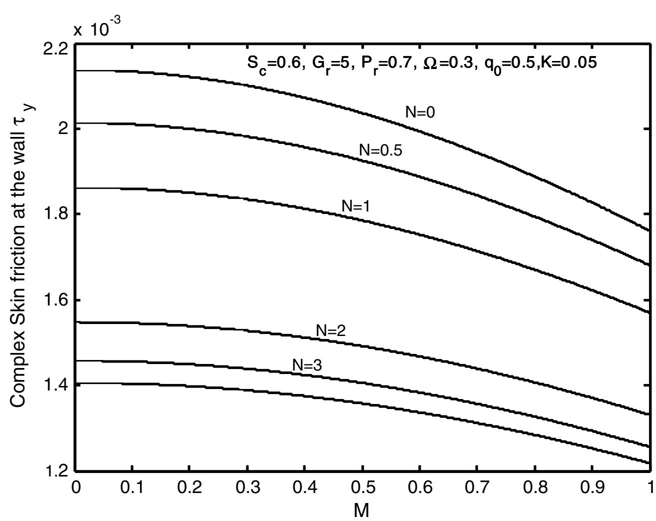

Fig. 10. Effect of radiation parameter on the imaginary part of the skin friction when $\omega < 2\Omega$ and $t = 0.03$.

the plate. However, we observe an increase in the temperature and the temperature boundary layer as the time parameter increases. Fig. 2 depicts the effects of chemical reaction and the Schmidt number on the species concentration. Again the feature is an exponential decrease away from the plate. As in Muthucumaraswamy and Ganesan [13], we take the Schmidt number $Sc = 0.24$ and 0.6 as would correspond to hydrogen and water vapour, respectively and observe that as the Schmidt number Sc is increased the concentration and the concentration boundary layer are both seen to decrease, whereas increase in the reaction rate constant k_r is accompanied by negligible change in the concentration and this is attributable to the effect of radiation on the flow.

The temperature and the species concentration are coupled to the velocity via the free convection parameters G_r and G_c, as seen in Eq. (11), and the effect of material parameters on the velocity are depicted in Figs. 3–7 (for cooling of the plate), and Fig. 8 (for heating of the plate) by free convection currents. Fig. 3 depicts the effect of Prandtl number on the flow velocity. We observe a decrease in the velocity as the Prandtl number increases due to the decrease in the temperature buoyancy effect which also leads to a decrease in the velocity boundary layer. From Figs. 4–6 we observe that increase in the Schmidt number or the free convection parameter or the radiation parameter leads to an increase in the velocity when the other parameters are fixed, for cooling of the plate by free convection currents $G_r > 0$). From Fig. 7 we observe that beyond about $t = 12$ the flow attains steady state conditions with the flow velocity increasing with increase in the time parameter t up to this point. From Fig. 8 we observe that besides the reverse flow, increase in the radiation parameter leads to an increase in the velocity. These observations are in good agreement with those of [14] and Chamkha [18] where these are attributed to the thermal buoyancy effect. Further in the absence of a magnetic field and mass transfer, we observe good agreement between this study and that of Bestman and Adjepong [1].

Figs. 9 and 10 show the effect of radiation on the complex skin friction τ_x and τ_y, where we observe an increase in the real part of the complex skin friction τ_x as the radiation parameter increases, and a decrease in the imaginary part of the complex skin friction as the radiation parameter increases. Further, from Fig. 7 we observe that the velocity increases as the time parameter t, increases, thus tending towards steady state conditions.

4. Conclusions

In this study we have examined the governing equations for unsteady hydro-magnetic natural convection heat and mass transfer flow of a rotating Boussinesq fluid past a vertical porous plate in the presence of radiative heat transfer. Employing Laplace transfer technique, the leading equations are solved analytically in the complex plane. We present results to illustrate the flow characteristics for the velocity and temperature fields as well as the skin friction and show how the flow fields are influenced by the material parameters of the flow problem. We can conclude from these results that the radiation indeed affects the temperature and therefore the velocity hence the skin friction. As the radiation parameter is increased it is accompanied by a decrease in the temperature and an increase in velocity, when the plate is cooled by convection currents, $G_r > 0$. Increase in the Schmidt number results in a decrease in the concentration, while increase in the rate constant results in negligible change in the concentration. The flow velocity approaches steady state conditions as the time parameter t, is increased to about 12 (Fig. 8).

In this part of our study we have emphasized only the real part of the complex fields. FORTRAN programming, for instance, can be employed to discuss the imaginary part of the complex fields. We hope to do that in a further study.

Acknowledgements

The contribution of two unknown referees is gratefully acknowledged.

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